Modeling of Kinetics of Air Entrainment in Water Produced by Vertically Falling Water Flow

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This study analyzes the process of air entrainment in water caused by vertically falling water flow in the free water surface. The new kinetic model of air entrainment in water was developed. This model includes the process of air entrainment, as well as air removal, water sputtering and resorption. For the experimental part of this study a new method based on digital image processing was developed. Theoretical and experimental methods were used for determining air concentration and its distribution in water below the air-water interface. A new presented mathematical model of air entrainment process allows determining of air bubbles and water droplets concentrations distribution. The obtained theoretical and experimental results were in good agreement.

Keywords: vertical water flow, entrainment, rise, sputtering, concentration distribution, air, water.

1. INTRODUCTION

A vertically falling water jet impingement in a quiescent water vessel creates an air entrainment in water. The air entrainment in the water also appears when the water moves through wears of hydropower stations or through river rifts. Waves in large open water bodies such as reservoirs, lakes, seas and oceans also stimulate air entrainment. The air entrainment in the water is an integrate part of the water aeration process, that on its own turn leads to the water purification.

The entrainment of gases in liquid trough multiphase flows is widely used in various fields of science, industry and environmental engineering. In chemistry bubbly flows are used to improve liquids mixing processes [1 – 5]. The air entrainment in the water by plunging the water jet has a potential application in many technological processes, for example, in the waste water treatment [6 – 10]. The water aeration processes are important tools of environment engineering because they highly influence the process of self-purification in rivers and other open reservoirs of the water [11 – 16]. One of the important factors of liquid aeration is a mixture of air bubbles and water droplets. This phenomenon was investigated by Chanson, Hoque, Karwa et. al. [17 – 19].

Understanding of mechanism of the air bubbles and the water droplets mixing produced by the water jet impinging on a free water surface has its practical importance, also, for the effective modelling of heat and mass transfer processes because in both cases the problem deals with two phase systems [20 – 24].

The goal of this paper is to develop mathematical model that allows a determination and prediction of air bubbles and water droplets concentration and its distribution in the zone of “white water” and verify it experimentally.

2. KINETIC MODEL

Due to the vertical water flow falling into the free water surface the water mixing occurs in the air-water interface. As a result of this, mixing air is entrained in the deeper water layers. The air-water mixture exists not only under the air-water interface but also above this interface because under water surface the air-water mixture blows up, the air bubbles rise up and are removed from the water. The falling water can also knock out the water sprays that can evaporate or fall down on the water surface. The fallen sprays can knock out new water sprays or be absorbed by the water. For description of all these processes we have developed a mathematical model. The model has significant relevance to development and improvement of aeration process and can be used for practical purposes.

The proposed theoretical model is one-dimensional entrainment of air at the free-falling jet.

All air entrainment zone we split up into $K$ monolayers that can be filled with air, water or sum of them. Normalized filling function of $j$-th monolayer was introduced:

$$
\phi_{j}^{(1)} = \phi_{j}^{(1)} + \phi_{j}^{(2)}, \text{ when } j = 1, 2, ..., K - 1, K, K + 1, \tag{1}
$$

Where $\phi_{j}^{(1)}$ and $\phi_{j}^{(2)}$ are functions of the monolayer filling with water and air respectively. Filling of $K$-th monolayer with air or water is continually varying. Rate of this functions $\phi_{j}^{(1)}$, $\phi_{j}^{(2)}$ can be described as a sum of equations of entrainment, removal, sputtering, evaporation and resorption rate:
where subscripts \( en \) – entrainment, \( el \) – elimination, \( sp+ev \) – sputtering and evaporation, \( resorb \) – resorption.

Solutions of Eq. (2) are concentrations of air bubbles and water droplets due to turbulent flows produced by free vertical water jet hitting the free water surface.

Let us discuss each process separately.

### 2.1. Entrainment and removal

Assuming that in \( K \)-th monolayer the rate of air entrainment in the water is proportional to the falling water flow \( i_{f}^{(K)} \) the rate of filling functions can be written as follow:

\[
\begin{align*}
\frac{d\varphi_{1}^{(j)}}{dt}_{en} &= i_{f}^{(j)} \cdot \beta, \quad j = 1, 2, \ldots, K - 1, K, K + 1; \\
\frac{d\varphi_{2}^{(j)}}{dt}_{en} &= i_{f}^{(j)} \cdot F_{R_{i}} \cdot \beta
\end{align*}
\]

where \( F_{R_{i}} \) – dimensionless impingement Froude number [26]. Usually, the air entrapped into depth is distributed according to Gaussian law [25]:

\[
i_{f}^{(j)} = i_{o} \cdot \exp \left[ - \frac{1}{2} \left( \frac{K \cdot dd - R_{p}}{\Delta R_{p}} \right)^{2} \right],
\]

where \( i_{o} \) (s\(^{-1}\)) is the total water jet flux, \( dd \) (m) is the mean value of thickness of the monolayer, \( R_{p} \) (m) is the average penetration of depth of air and \( \Delta R_{p} \) (m) is the standard deviation, \( \beta \) – dimensionless coefficient, which is equal to the probability of finding entrapped air or water in the varied monolayer, \( v_{i} \) – water flow velocity in incident flux zone (m/s), \( g \) – acceleration due to gravity (m/s\(^2\)), \( d_{l} \) – is the jet impact diameter (m).

The maximum depth of the air entrainment into the water can be calculated as follow [27]:

\[
z_{p} = \frac{1.2 \cdot v_{o}^{0.77} \cdot d_{o}^{0.625}}{k_{l, h}^{0.084}},
\]

where \( z_{p} \) is the maximum penetration depth of air, \( d_{o} \) is the jet nozzle diameter, \( h \) is the distance from nozzle tip to the water surface, \( k_{l, h} = 1.0 \) is the overall mass transfer coefficient (s\(^{-1}\)).

Solving equation (3), we have assumed that the top layer filled with water monolayer is \( m \)-th, and the \( K \)-th – monolayer is the deepest. Accordingly, we have chosen the following boundary conditions when \( t = 0 \):

\[
\varphi_{1}^{(j)}(0) = 0, \quad \text{if} \quad m \leq j \leq K \quad \text{and} \quad \varphi_{1}^{(j)}(0) = 0, \quad \text{if} \quad 1 \leq j < m \quad \text{and} \quad \varphi_{1}(0) = 0, \quad \text{if} \quad m \leq j \leq K \quad \text{and} \quad \varphi_{1}^{(j)}(0) = 1, \quad \text{if} \quad 1 \leq j < m.
\]

From equation (3) it is seen that \( \varphi_{1}^{(j)} \), \( \varphi_{2}^{(j)} \) grows infinitely during the entrapment process. Therefore, in order to keep the condition \( \varphi^{(j)} = 1 \), it is necessary to perform renormalization of monolayers. The \( K \)-th layer’s air and water overall total amount, which exceeds one (>1) i.e. \( \varphi^{(j)} = \varphi^{(j)} - 1 \) is displaced into the \( (K - 1) \) monolayer. Fractions of the air \( f_{2}^{(j)} \) and water \( f_{1}^{(j)} \) taking place in the \( K \)-th monolayer can be calculated as follows:

\[
\begin{align*}
f_{2}^{(j)} &= \frac{\varphi_{2}^{(j)}(0)}{\varphi_{1}^{(j)}(0) + \varphi_{2}^{(j)}}; \\
f_{1}^{(j)} &= \frac{\varphi_{1}^{(j)}(0)}{\varphi_{1}^{(j)}}.
\end{align*}
\]

Total amount of the air and water displaced from \( K \)-th layer into \( (K - 1) \) layer must have the same ratio that it had in \( K \)-layer. Displaced parts of the air \( \varphi_{1}^{(K)} \) and water \( \varphi_{2}^{(K)} \) can be expressed with this equation:

\[
\begin{align*}
\varphi_{1}^{(j)}(j+1) &= \varphi_{1}^{(j)}(j) + \varphi_{2}^{(j)}(j), \\
\varphi_{2}^{(j)}(j+1) &= \varphi_{2}^{(j)}(j) + \varphi_{1}^{(j)}(j).
\end{align*}
\]

Evaluating equation (7), filling of the monolayer \( (K - 1) \) with the air and water can be recalculated as follow:

\[
\begin{align*}
\varphi_{2}^{(j)}(j+1) &= \varphi_{2}^{(j)}(j) + \varphi_{2}^{(j)}(j), \\
\varphi_{1}^{(j)}(j+1) &= \varphi_{1}^{(j)}(j) + \varphi_{1}^{(j)}(j).
\end{align*}
\]

The air bubbles entrained in the water under acting of Archimedes force raises from the deeper layers to the water surface. Part of the bubbles joins upper monolayers bubbles and increases in volume. This process is analogous to process of liquids boiling where the air bubbles increases going up to the water surface. Consequently, the ensuing “empty spaces” are replaced with water. Let us suppose that air eliminated is defined as \( \varphi_{2}^{(k)} - \varphi_{2}^{(k+1)} \). The rate of change filling functions due to the air bubbles rise can be expressed as:

\[
\begin{align*}
\frac{d\varphi_{1}^{(j)}}{dt}_{el} &= \alpha \varphi_{2}^{(j+1)} - \alpha \varphi_{2}^{(j)} \\
\frac{d\varphi_{2}^{(j)}}{dt}_{el} &= \alpha \varphi_{2}^{(j+1)} - \alpha \varphi_{2}^{(j)}
\end{align*}
\]

where \( \alpha \) is the air bubbles elimination proportionality coefficient, (s\(^{-1}\)), subscript \( el \) – elimination.

### 2.2. Sputtering and evaporation

Let us suppose that \( w_{H} \) is the frequency probability of the water droplets sputtered from the surface by the flux \( i_{H} \). Usually, the water droplets can be sputtered only from monolayer covered with water. The rate of change filling functions due to sputtering can be expressed as:

\[
\begin{align*}
\frac{d\varphi_{1}^{(j)}}{dt}_{sp+ev} &= \left\{ \begin{align*}
w_{H} [\varphi_{1}^{(j)} - \varphi_{2}^{(j)}], & \text{if} \quad \varphi_{1}^{(j)} \geq \varphi_{2}^{(j)} \\
0, & \text{if} \quad \varphi_{1}^{(j)} < \varphi_{2}^{(j)}
\end{align*} \right. \\
\frac{d\varphi_{2}^{(j)}}{dt}_{sp+ev} &= \frac{d\varphi_{2}^{(j)}}{dt}
\end{align*}
\]

}\]
where \( w_H = w_t + w_G = Y \cdot i_H + A \exp(-Q / k \cdot T) \), \( w_t \) is the frequency of probability of sputtering, \( w_G \) is the frequency of probability of evaporation, \( Y \) is the dimensionless coefficient of sputtering, \( k \) is the Boltzmann constant \((1/K)\), \( T \) is the temperature \((K)\), \( A \) is the evaporation coefficient, subscript \( sp+ev \) denotes sputtering and evaporation.

2.3. Resorption

Sputtered water droplets can be resorbed back to the surface. Probabilities of resorbed water droplets sticking to the air or to the water are \( \alpha_{ho} \) and \( \alpha_{hh} \), respectively. Following similar considerations used in deriving equation (10), the processes of resorption can be expressed as:

\[
\begin{align*}
\frac{d\phi_t}{dt}_{\text{resorb}} & = \frac{d\phi_t}{dt} \chi_{Hh} \chi_{Ha} \alpha_{h}^2, \quad \text{if } \phi_t > \phi_h^2, \\
\frac{d\phi_h}{dt}_{\text{resorb}} & = \frac{d\phi_t}{dt} \chi_{Hh} \chi_{Ha} \alpha_{h}^2, \quad \text{if } \phi_t < \phi_h^2,
\end{align*}
\]

\( j = 1, 2, K, (11) \)

where \( \chi_{Hh} = \alpha_{hh} \cdot i_k, \chi_{HA} = \alpha_{ho} \cdot i_k, i_k = B \cdot i_{sp} + i_k \), \( i_{sp} = w_H \cdot (\phi_1^{(k)} - \phi_1^{(k-1)}) \), \( \alpha_{hh} \) is the water droplets sticking to the water probability, \( \alpha_{ho} \) is the water droplets sticking to the air probability, \( i_k \) is the flow of absorbed water droplets, \( B \) is the \((0 \leq B \leq 1)\) coefficient which defines the fraction of sputtered water returning to the surface. The first term in equation (11) describes the falling water droplet sticking to the water of \( K \)-th monolayer with the probability \( \alpha_{hh} \), the second term describes the falling water droplet sticking to the air with a probability \( \alpha_{ho} \).

3. RESULTS AND DISCUSSION

\( K \) monolayers are formed after air entrainment. These monolayers are filled with the air, air-water or water. Monolayer \( K = 60 \) was taken as a limitary monolayer. The last monolayer filled with the air and water was \( m \)-th and \( m+1 \) was filled only with water. For modeling according to equations (2) till (11) at time \( t = 0 \), such initial and boundary conditions were taken: \( \phi_2^{(K)}(0) = 1, \phi_1^{(K)}(0) = 0 \), when \( K < 60 \) and \( \phi_2^{(K)}(0) = 0, \phi_1^{(K)}(0) = 1 \), \( \phi_m(t) = \phi_{m+1}(t) \) when \( K \geq 60 \). Solving equations (2–11) initially was taken, that falling water flow in all monolayers is: \( i_H = i_0 = \text{const.} \). Then for different Froude numbers monolayers filling with air decrease from one to zero (Fig. 1).

As can be seen from Fig. 1, the 60-th monolayer is the air - water interface monolayer. Filling of monolayers with the water is reciprocal process therefore it changes from zero to one.

For calculating the air and water concentrations in the monolayers according to equation (2) it was taken that the probability of the water droplets sticking to the water is \( \alpha_{hh} = 0.95 \), and for the air – \( \alpha_{ho} = 0.0000009 \), sputtering coefficient \( Y = 0.00095 \).

Vertically falling water jet was used for stimulation of the air entrainment. During the experiment water flow fell into a vessel. The air temperature \( T_{air} = 20 \, ^\circ\text{C} \), the water temperature \( T_w = 18 \, ^\circ\text{C} \), the water density \( \rho_w = 1000 \, \text{kg/m}^3 \), water’s surface tension \( \sigma_w = 0.073 \, \text{N/m} \), the pressure \( P = 101.3 \, \text{kPa} \). Detailed description of the experiment is presented in [28]. The experiment has shown that distribution of the concentration of the air entrained corresponds to the Gaussian distribution (Fig. 2).

As can be seen from Fig. 2 the image representing the experiment consists of three parts. On the left side (Fig. 2, a) it is a real image of air bubbles and water droplets mixture. The central part of figure (Fig. 2, b) represents modified image of the air entrainment in the water. Computer program code analyzes the received images and presents data. For digital image processing the images library Python 2.7 was used [29]. The black digital image is the modified image of the air bubbles and the white image is of the water droplets. On the right side of the figure (Fig. 2, c) the change of concentration of the air entrained in depth and width is represented. In addition, the Figure 2, c, shows that air entrainment in deeper layers changes according to the Gaussian law. Therefore, for solving equations (4a) it was taken that falling water flow \( i_H \) is distributed in the monolayers to depth according to the Gaussian distribution.

![Fig. 1. Air and water distribution in monolayers for different Froude numbers Fr = 11.7, 18.1, 23.4. Modeling was done according equations (2)–(11) taking that in all monolayers \( i_H = i_0 = 0.095 \).](image)

![Fig. 2. Change of air entrainment; a – experimental real image, b – modified image, c – air concentration dependence on depth](image)

Fig.3 represents the change of dependence on the monolayers of filling with the air and water for different Froude numbers \( Fr = 11.7, 18.1, 23.4 \). The average penetration depth \( R_p \) is calculated with empirical eq. (4b). As can be seen from Fig. 3 that the layer separating air and water is layer \( K = 60 \). Lines 1, 2, 3 show monolayers’
covered with water, and dashed lines 1, 2, 3 – with air (Fig. 3). Initially, the concentration of the air entrained increases till the maximum, then it decreases till reaches the air and water boundary and its value becomes zero. This process with water is reciprocal. As can be seen from inserted diagram (Fig. 3) the function $\varphi_{\text{air max}}$ slowly grows with increasing of the Froude number $Fr_1$. Also, maxima of concentrations of the air entrained slowly increases according to $\varphi_{\text{air max}} \propto 0.3 \ln(Fr_1) - 0.58$. Thus, the inserted graph is a natural logarithm function.

Curve 3 (Fig. 3) corresponds Froude number $Fr_1 = 11.7$, maximum at $K = 165$, curve 2 – $Fr_1 = 18.1$ (maximum at $K = 195$), and curve 1 – $Fr_1 = 23.4$ (maximum at $K = 210$). Accordingly, with increasing Froude number the amount of air entrained also increases. The average penetration depth of the air can be calculated according equation (4, b). This model was verified by the experiment taking $Fr_1 = 11.7; 18.1; 23.4$, nozzle diameter $d_0 = 0.004$ m, height of the falling flow $h = 0.25$ m, 0.5 m and 0.75 m. The results received from the experiment are presented in Fig. 4.

As can be seen from Fig. 4, with Froude number increasing the maximal air entrainment in depth also increases and position of the maximum in the monolayer changes. Curve 1 – corresponds to the concentration of the air entrained, when $v_1 = 2.2$ m/s, $d_0 = 0.004$ m (diameter of nozzle), $d_1 = 0.0036$ m, $Fr_1 = 11.7$. Curve 2 – corresponds to concentration, when $v_1 = 3.1$ m/s, $d_0 = 0.004$ m, $d_1 = 0.003$ m, $Fr_1 = 18.1$. Curve 3 – corresponds to the concentration, when $v_1 = 3.8$ m/s, $d_0 = 0.004$ m, $d_1 = 0.0027$ m, $Fr_1 = 23.41$ (Fig. 4). This experiment shows that with Froude number increasing air entrainment in depth changes i.e. increases, when velocity $v_1$ increases because $Fr_1 \sim v_1$. For modeling of the entrainment processes with equation (2) the influence of each parameter on the air and water concentration’s changes should be considered separately. The influence of Froude number on the air entrainment was discussed above. Therefore, the influence of the falling water flow $i_l$ (Fig. 5) should be also discussed.

Thus, for the solving equation we were taken that 60-th the layer is the air and water boundary layer it is worth to discuss the processes acting above monolayer 60-th. Change of the air concentration was signed with letter “a” and number (dashed lines in Fig. 5).

For modeling with equation (2) was taken that $Fr_1 = 30$ and the falling flow changes from $i_l = 0.95$ to $i_l = 0.11$. The highest concentration of the air entrained was received when $i_l = 0.95$, $\varphi_{\text{air max}} = 0.6$, $K = 200$ (Fig. 5, 1a).

As can be seen in inclusion (Fig. 5) the concentrations of maxima of the air entrained dependence on $i_l$ is the linear function. Another important parameter is $w_H$. When $w_H = 0.0$, then also $w_S = 0.0$ and $w_G = 0.0$. In this case, there exist only the air entrainment and removal processes, the water droplets are not knocked out from the water surface. The modeling gives that the solutions of equations (10) and (11) are equal to zero. When during the falling flow $w_H \neq 0.0$ and $w_G = 0.0$, the concentration of the air entrained in the water increases with increasing $w_H$ (Fig. 6). The graphic inserted (Fig. 6) shows the dependence of the maxima of the air entrained on parameter $w_H$. When the parameter $w_H$ increases, then function of maxima of the air entrained is linear and can be described as: $\varphi_{\text{air max}} \propto 0.58 \cdot w_H + 0.1$.

The parameter that must be evaluated is $w_S$. When $w_S = 0.0$, there is no water droplets sputtering, only the evaporation exists. Fig. 7 represents dependences of the air entrainment, when $T_{\text{air}} = 20^\circ$C. As can be seen from Fig. 7,
when \(w_s = 0.0\), the the curve of the concentration of the air entrained coincides with the curve of Fig. 8 when temperature \(T = 20^\circ\text{C}\). With frequency of a probability of sputtering increasing concentration of the air entrained increases also. The inserted diagram (Fig. 7) shows that the dependence \(\varphi_{\text{max}}^{\text{air}} \propto 0.12 \cdot w_s + 0.25\) is linear, i.e. function \(\varphi_{\text{max}}\) increases linear with increasing of \(w_s\).

**4. CONCLUSIONS**

1. A new theoretical and experimental technique presented allowed determination of the air entrained concentration distribution below air-water interface. Theoretical and experimental results showed good coincidence.

2. Theoretical model gives that with increase of Froude number \(F_{\text{Fr}}\), the maxima of the air entrained concentrations slowly increase according to a natural logarithm function.

3. Mathematical modeling showed that change (increase or decrease) of the falling water flow \(i_H\) causes linear change of the air entrained concentration’s maxima according to linear function.

4. Sputtering of the water droplets depends on parameter \(w_H\). The change of \(w_H\) leads to the liner change of maximum of the air entrained concentration according to linear function.

5. The air temperature influences processes of the evaporation and resorption. Theoretically was determined that increasing the air temperature, the maxima of the air entrained concentrations decrease according to a natural exponential function.

**REFERENCES**


